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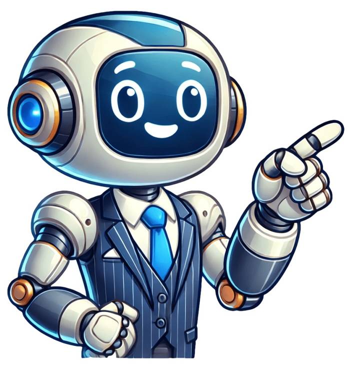

We know that the absolute value of a number is always positive (or zero). We can see this same result reflected in the graph of the absolute value parent function y = | x |. All of the graph's y-values will be positive (or zero). The graph of the absolute value parent function is composed of two linear "pieces" joined together at a common vertex (the origin). The graph of such absolute value functions generally takes the shape of a V, or an up-side-down V. Notice that the graph is symmetric about the y-axis. Linear "pieces" will appear in the equation of the absolute value function in the following manner: y = | mx + b | + c where the vertex is (-b/m, c) and the axis of symmetry is x = -b/m. Find your Math Personality! 28 December 2020 Read time: 5 minutes We have already understood in detail about the absolute function in the blog about the absolute value function. Let us move on to a major aspect of solving absolute value equations which is drawing the necessary graph, looking at the intercepts and vertex. Hence, graphing absolute value functions is an important topic which we will reduce to a step by step easy process. Absolute value graph-PDF Cuemath's learning platform has carefully designed simulations to help the student visualize concepts. This blog will help us understand the absolute value graph. Here is a downloadable PDF to explore more. Absolute value graph-PDF Download Let us look at the most basic absolute value function graph, \[y = |x|\] Most of the absolute value function graphs will have a somewhat similar shape, a V-like structure with a vertex. Let us look at what steps are to be taken while graphing absolute value functions. The following steps will be useful in graphing absolute value functions. How to graph an absolute value function? Step 1: Before graphing any absolute value function, first we have to graph the absolute value parent function: \[y = |x|\] Its vertex is (0,0) Let us take some random values for x. \[\begin{align}x &= - 3 \quad\rightarrow\quad y = |-3| = 3 \quad\rightarrow\quad (-3, 3)\\x &= - 2 \quad\rightarrow\quad y = |-2| = 2 \quad\rightarrow\quad (-3, 3)\\x &= - 1 \quad\rightarrow\quad y = |-1| = 1 \quad\rightarrow\quad (-3, 3)\\x &= 0 \quad\rightarrow\quad y = |0| = 0 \quad\rightarrow\quad (0, 0)\\x &= 1 \quad\rightarrow\quad y = |1| = 1 \quad\rightarrow\quad (1, 1)\\x &= 2 \quad\rightarrow\quad y = |2| = 2 \quad\rightarrow\quad (2, 2)\\x &= 3 \quad\rightarrow\quad y = |3| = 3 \quad\rightarrow\quad (3, 3)\end{align}\] If we plot these points on the graph sheet, we will get a graph as given below. When we look at the above graph, clearly the vertex is (0, 0) Step 2: Write the given absolute value function as \(y - k = |x - h|\) Step 3: To get the vertex of the absolute value function above, equate (x - h) and (y - k) to zero, That is, \(x - h = 0\) and \(y - k = 0\) \(x = h\) and \(y = k\) Therefore, the vertex is \((h, k)\) According to the vertex, we have to shift the above graph. Note: If we have negative signs in front of absolute signs, we have to flip the curve over. \[y = - |x|\] These steps should be kept in mind in graphing absolute value function. We now take more complex absolute function examples. Graphing Absolute Value Functions Solved Examples Graphing absolute value function given below. \[y = |x - 1|\] Solution : The given absolute value function is in the form : \[y - k = |x - h|\] That is, \[y = |x - 1|\] To get the vertex, equate (x - 1) and y to zero. \(x - 1 = 0\) and \(y = 0\) \(x = 1\) and \(y = 0\) Therefore, the vertex is So, the absolute value graph of the given function is Absolute value graph of: \[y = |x - 1| - 2\] Solution : Write the given absolute value function in the form : \[y - h = |x - h|\] That is, \[y = |x - 1| - 2\] Add 2 to each side. \[y + 2 = |x - 1|\] To get the vertex, equate (x - 1) and (y + 2) to zero. \(x - 1 = 0\) and \(y + 2 = 0\) \(x = 1\) and \(y = -2\) Therefore, the vertex is So, the absolute value graph of the given function is Graph the absolute value function given below. \[y = |x + 3| + 3\] Solution : Write the given absolute value function in the form : \[y - h = |x - h|\] That is, \[y = |x + 3| + 3\] Subtract 3 from each side. \[y - 3 = |x + 3|\] To get the vertex, equate (x + 3) and (y - 3) to zero. \(x + 3 = 0\) and \(y - 3 = 0\) \(x = -3\) and \(y = 3\) Therefore, the vertex is So, the absolute value graph of the given absolute value function is How to graph absolute value: \[y = |x - 2|\] Solution : The given absolute value function is in the form : \[y - k = |x - h|\] That is, \[y = |x - 2|\] To get the vertex, equate (x - 2) and y to zero. \(x - 2 = 0\) and \(y = 0\) \(x = 2\) and \(y = 0\) Therefore, the vertex is So, the absolute value graph of the given absolute value function is How to graph absolute value: \[y = |x + 4| + 3\] Solution : Write the given absolute value function in the form : \[y - h = |x - h|\] That is, \[y = |x + 4| + 3\] Subtract 3 from each side. \[y - 3 = |x + 4|\] To get the vertex, equate (x + 4) and (y - 3) to zero. \(x + 4 = 0\) and \(y - 3 = 0\) \(x = -4\) and \(y = 3\) Therefore, the vertex is So, the absolute value graph of the given absolute value function is How to graph absolute value: \[y = |x - 4| - 4\] Solution : Write the given absolute value function in the form : \[y - h = |x -h|\] That is, \[y = |x - 4| - 4\] Add 4 to each side. \[y + 4 = |x - 4|\] To get the vertex, equate (x - 4) and (y + 4) to zero. \(x - 4 = 0\) and \(y + 4 = 0\) \(x = 4\) and \(y = -4\) Therefore, the vertex is So, the absolute value graph of the given absolute value function is How to graph absolute value equations \[y = -|x - 2| - 2\] Solution : Write the given absolute value function in the form \[y - h = |x - h|\] That is, \[y = -|x - 2| - 2\] Add 2 to each side. \[y + 2 = -|x - 2|\] To get the vertex, equate (x - 2) and (y + 2) to zero. \(x - 2 = 0\) and \(y + 2 = 0\) \(x = 2\) and \(y = -2\) Therefore, the vertex is Because there is a negative sign in front of the absolute sign, we have to flip the curve over. So, the absolute value graph of the given absolute value function is How to graph absolute value equations \[y = -|x - 4|\] Solution : The given absolute value function is in the form : \[y - k = |x - h|\] That is, \[y = -|x - 4|\] To get the vertex, equate (x - 4) and y to zero. \(x - 4 = 0\) and \(y = 0\) \(x = 4\) and \(y = 0\) Therefore, the vertex is Because there is a negative sign in front of the absolute sign, we have to flip the curve over. This is an important function transformation. So, the absolute value graph of the given absolute value function is Graph the absolute value function given below. \[y = -|x| + 2\] Solution : Write the given absolute value function in the form : \[y - h = |x - h|\] That is, \[y = -|x| + 2\] Subtract 2 from each side. \[y - 2 = -|x|\] To get the vertex, equate x and (y - 2) to zero. \(x = 0\) and \(y - 2 = 0\) \(x = 0\) and \(y = 2\) Therefore, the vertex is Because there is a negative sign in front of the absolute sign, we have to flip the curve over. Hence, the graph of the given absolute value function is Graphing absolute value function: \[y = -|x + 1| + 3\] Solution : Write the given absolute value function in the form : \[y - h = |x - h|\] That is, \[y = -|x + 1| + 3\] Subtract 3 from each side. \[y - 3 = -|x + 1|\] To get the vertex, equate (x + 1) and (y - 3) to zero. \(x + 1 = 0\) and \(y - 3 = 0\) \(x = -1\) and \(y = 3\) Therefore, the vertex is Because there is a negative sign in front of the absolute sign, we have to flip the curve over and apply the function transformation. So, the absolute value graph of the given absolute value function is Graphing absolute value equations \[y = -|x| + 4\] Solution : Write the given absolute value function in the form \[y - h = |x - h|\] That is, \[y = -|x| + 4\] Subtract 4 from each side. \[y - 4 = -|x|\] To get the vertex, equate x and (y - 4) to zero. \(x = 0\) and \(y - 4 = 0\) \(x = 0\) and \(y = 4\) Therefore, the vertex is Because there is a negative sign in front of the absolute sign, we have to flip the curve over. So, the absolute value graph of the given absolute value function is Graphing absolute value equations. \[y = -|x + 1| - 1\] Solution : Write the given absolute value function in the form : \[y - h = |x - h|\] That is, \[y = -|x + 1| - 1\] Add 1 to each side. \[y + 1 = -|x + 1|\] To get the vertex, equate (x + 1) and (y + 1) to zero. \(x + 1 = 0\) and \(y + 1 = 0\) \(x = -1\) and \(y = -1\) Therefore, the vertex is Because there is a negative sign in front of the absolute sign, we have to flip the curve over. So, the absolute value graph of the given absolute value function is Vertical and Horizontal Function Transformations Have you ever tried to draw a picture of a rabbit, or cat, or animal? Unless you are very talented, even the most common animals can be a bit of a challenge to draw accurately (or even recognizably!). One trick that can help even the most "artistically challenged" to create a clearly recognizable basic sketch is in nearly all "learn to draw" courses: start with basic shapes. By starting your sketch with simple circles, ellipses, etc. the basic outline of the more complex figure is easily arrived at, then details can be added as necessary, but the figure is already recognizable for what it is. The same trick works when graphing absolute value equations. By learning the basic shapes of different types of function graphs, and then adjusting the graphs with different types of transformations, even complex graphs can be sketched rather easily. This section will focus on two particular types of transformations: vertical shifts and horizontal shifts. We can express the application of vertical shifts this way: Note : For any function \(f(x)\), the function \(g(x) = f(x) + c\) has a graph that is the same as \(f(x),\) shifted c units vertically. If c is positive, the graph is shifted up. If c is negative, the graph is shifted down. Note: Adding a positive number after the x outside the parentheses shifts the graph up, adding a negative (or subtracting) shifts the graph down. We can express the application of horizontal shifts this way: Note: given a function \(f(x)\), and a constant a > 0, the function \(g(x) = f(x - a)\) represents a horizontal shift of a unit to the right from \(f(x).\) The function \(h(x) = f(x + a)\) represents a horizontal shift of a unit to the left. Note: Adding a positive number after the x inside the parentheses shifts the graph left, adding a negative (or subtracting) shifts the graph right. Below is the graphing of absolute value equationThe graph of \(y= |x|\) has been shifted right 3 units, vertically stretched by a factor of 2, and shifted up 4 units. This means that the corner point is located atfor this transformed function. Writing an Equation for an Absolute Value Function Given a Graph. We also notice that the graph appears vertically stretched, because the width of the final graph on a horizontal line is not equal to 2 times the vertical distance from the corner to this line, as it would be for an unstretched absolute value function. Instead, the width is equal to 1 times the vertical distance as shown in (Figure). From this information we can write the equation Analysis Note that these equations are algebraically equivalent—the stretch for an absolute value function can be written interchangeably as a vertical or horizontal stretch or compression. If we couldn't observe the stretch of the function from the graphs, could we Write an equation for the function graphed in (Figure). The basic absolute value function changes direction at the origin, so this graph has been shifted to the right 3 units and down 2 units from the basic toolkit function. See (Figure). algebraically determine it? Yes. If we are unable to determine the stretch based on the width of the graph \(f(x)=a|x-3|-2\) Now substituting in the point \((1, 2)\) \[\begin{align}2&=a|1-3|-2\\4&=2a\\a&=2\end{align}\] Write the equation for the absolute value function that is horizontally shifted left 2 units, is vertically flipped, and vertically shifted up 3 units. \[f(x)=-|x+2|+2\] Do the graphs of absolute value functions always intersect the vertical axis? The horizontal axis? Yes, they always intersect the vertical axis. The graph of an absolute value function will intersect the vertical axis when the input is zero. No, they do not always intersect the horizontal axis. The graph may or may not intersect the horizontal axis, depending on how the graph has been shifted and reflected. It is possible for the absolute value function to intersect the horizontal axis at zero, one, or two points (see (Figure)). (a) The absolute value function does not intersect the horizontal axis. (b) The absolute value function intersects the horizontal axis at one point. (c) The absolute value function intersects the horizontal axis at two points. The process of graphing absolute value equations can be reduced to some specific steps which help develop any kind of absolute value graph. These can be achieved by first starting with the parent absolute value function, then shifting the graph according to function transformations, flip graph if necessary and even may have to compress or decompress the graph. Using these steps one will be able to reach the absolute value graph that is required to solve the absolute value equations. About Cuemath Cuemath, a student-friendly mathematics and coding platform, conducts regular Online Classes for academics and skill-development, and their Mental Math App, on both iOS and Android, is a one-stop solution for kids to develop multiple skills. Understand the Cuemath Fee structure and sign up for a free trial. In order to continue enjoying our site, we ask that you confirm your identity as a human. Thank you very much for your cooperation. c - To sketch the graph of \(f(x) = |(x - 2)^2 - 4| \), we first sketch the graph of \(y = (x - 2)^2 - 4 \) and then take the absolute value of\(y \). The graph of \(y = (x - 2)^2 - 4 \) is a parabola with vertex at \((2,-4) \), x intercepts at \((0 , 0) \) and \((4 , 0) \) and a y intercept \((0 , 0) \). (see graph below) The graph of \(f \) is given by reflecting on the x axis part of the graph of \(y = (x - 2)^2 - 4 \) for which \(y \) is negative. (see graph below). Figure \(\PageIndex{ 1 } \) Distances in deep space can be measured in all directions. As such, it is useful to consider distance in terms of absolute values. (credit: "s58y"/Flickr)Until the 1920s, the so-called spiral nebulae were believed to be clouds of dust and gas in our own galaxy, some tens of thousands of light years away. Then, astronomer Edwin Hubble proved that these objects are galaxies in their own right, at distances of millions of light years. Today, astronomers can detect galaxies that are billions of light years away. Distances in the universe can be measured in all directions. As such, it is useful to consider distance as an absolute value function. In this section, we will investigate absolute value functions. Recall that in its basic form \(f(x)=| x |\), the absolute value function, is one of our toolkit functions. The absolute value function is commonly thought of as providing the distance the number is from zero on a number line. Algebraically, for whatever the input value is, the output is the value without regard to sign. The absolute value function is defined to be\[f(x) = |x| = \begin{cases} x, & \text{ if } x \geq 0 \\[6pt] -x, & \text{ if } x < 0 \\[6pt] \end{cases} onumber \] Describe all values \(x\) within or including a distance of 4 from the number 5. Solution We want the distance between \(x\) and 5 to be less than or equal to 4. We can draw a number line, such as the one in Figure \(\PageIndex{ 2 } \), to represent the condition to be satisfied. Figure \(\PageIndex{ 2 } \) The distance from \(x\) to 5 can be represented using the absolute value as \(| x−5 |.\) We want the values of \(x\) that satisfy the condition \(| x−5 | \leq 4.\) Note that\[\begin{array}{rclcrcl} −4 & \leq & x−5 & \quad & x−5 & \leq & 4 \\[6pt] 1 & \leq & x & \quad & x & \leq & 9. \\[6pt] \end{array} onumber \]So \(| x−5 | \leq 4\) is equivalent to \(1 \leq x \leq 9\). However, mathematicians generally prefer absolute value notation. Describe all values \(x\) within a distance of 3 from the number 2. Electrical parts, such as resistors and capacitors, come with specified values of their operating parameters: resistance, capacitance, etc. However, due to imprecision in manufacturing, the actual values of these parameters vary somewhat from piece to piece, even when they are supposed to be the same. The best that manufacturers can do is to try to guarantee that the variations will stay within a specified range, often \(\pm 1\%\), \(\pm 5\%\), or \(\pm 10\%\). Suppose we have a resistor rated at 680 ohms, \(\pm 5\%\). Use the absolute value function to express the range of possible values of the actual resistance. Solution 5% of 680 ohms is 34 ohms. The absolute value of the difference between the actual and nominal resistance should not exceed the stated variability, so, with the resistance \(R\) in ohms,\[| R−680 | \leq 34. onumber \] Students who score within 20 points of 80 will pass a test. Write this as a distance from 80 using absolute value notation. The most significant feature of the absolute value graph is the corner point at which the graph changes direction. This point is shown at the origin in Figure \(\PageIndex{ 3 } \). Figure \(\PageIndex{ 3 } \) Figure \(\PageIndex{ 4 } \) shows the graph of \(y=2| x–3 |+4.\) The graph of \(y=| x |\) has been shifted right 3 units, vertically stretched by a factor of 2, and shifted up 4 units. This means that the corner point is located at \((3,4)\) for this transformed function. Figure \(\PageIndex{ 4 } \) Write an equation for the function graphed in Figure \(\PageIndex{ 5 } \). Figure \(\PageIndex{ 5 } \) Solution The basic absolute value function changes direction at the origin, so this graph has been shifted to the right 3 units and down 2 units from the basic toolkit function. See Figure \(\PageIndex{ 6 } \). Figure \(\PageIndex{ 6 } \) We also notice that the graph appears vertically stretched, because the width of the final graph on a horizontal line is not equal to 2 times the vertical distance from the corner to this line, as it would be for an unstretched absolute value function. Instead, the width is equal to 1 times the vertical distance as shown in Figure \(\PageIndex{ 7 } \). Figure \(\PageIndex{ 7 } \) From this information we can write the equation as \(f(x)=2|x−3|−2\), treating the stretch as a vertical stretch. In Example \(\PageIndex{ 3 } \), we could have opted to treat the stretch as a horizontal compression. In that case, the function would have been \(f(x)=|2(x−3)|−2 \). Note that this equation is algebraically equivalent to what we arrived at in our solution. This result, that the stretch for an absolute value function can be written interchangeably as a vertical or horizontal stretch or compression, is not a rule for all functions - it just happens to work for absolute value (and linear) functions. Note also that if the vertical stretch factor is negative, there is also a reflection about the x-axis. If we couldn't observe the stretch of the function from the graphs, could we algebraically determine it? Yes. If we are unable to determine the stretch based on the width of the graph, we can solve for the stretch factor by putting in a known pair of values for \(x\) and \(f(x)\).\[f(x)=a|x−3|−2. onumber \]Now substituting in the point \((1, 2) \), we get\[\begin{array}{rrclcl} & 2 & = & a| 1−3 |−2 & \quad & \left(\text{substituting} \right) \\[6pt] \implies & 4 & = & 2a & \quad & \left(\text{simplifying and adding }2 \text{ to both sides} \right) \\[6pt] \implies & 2 & = & a & \quad & \left(\text{dividing both sides by }2 \right) \\[6pt] \end{array} onumber \] Write the equation for the absolute value function that is horizontally shifted left 2 units, is vertically flipped, and vertically shifted up 3 units. Do the graphs of absolute value functions always intersect the vertical axis? The horizontal axis? Yes, they always intersect the vertical axis. The graph of an absolute value function will intersect the vertical axis when the input is zero. No, they do not always intersect the horizontal axis. The graph may or may not intersect the horizontal axis, depending on how the graph has been shifted and reflected. It is possible for the absolute value function to intersect the horizontal axis at zero, one, or two points (see Figure \(\PageIndex{ 8 } \)). Figure \(\PageIndex{ 8(a) } \): The absolute value function does not intersect the horizontal axis. Figure \(\PageIndex{ 8(b) } \): The absolute value function intersects the horizontal axis at one point. Figure \(\PageIndex{ 8(c) } \): The absolute value function intersects the horizontal axis at two points. Is the absolute value function one-to-one? Now that we can graph an absolute value function, we will learn how to solve an absolute value equation. To solve an equation such as \(8=| 2x−6 |\), we notice that the absolute value will be equal to 8 if the quantity inside the absolute value is 8 or -8. This leads to two different equations we can solve independently.\[\begin{array}{rrclcrcl} & 2x−6 & = & 8 & \text{ or } & 2x−6 & = & −8 \\[6pt] \implies & 2x & = & 14 & & 2x & = & −2 \\[6pt] \implies & x &= &7 & & x & = & −1 \\[6pt] \end{array} onumber \]Knowing how to solve problems involving absolute value functions is useful. For example, we may need to identify numbers or points on a line that are at a specified distance from a given reference point. An absolute value equation is an equation in which the unknown variable appears within the absolute value bars. For example,\[\begin{array}{rcl} | x | & = & 4,\\[6pt] | 2x−1 | & = & 3, \text{ and}\\[6pt] | 5x+2 |−4 & = & 9. \\[6pt] \end{array} onumber \] To solve an absolute value equation, \(| \blacksquare | = k \), where \(\blacksquare \) is an algebraic expression involving a variable, we rely on the piecewise function definition of the absolute value. Namely,\[| \blacksquare |= \begin{cases} \blacksquare, & \text{ if } \blacksquare \geq 0 \\[6pt] −\blacksquare, & \text{ if } \blacksquare